Markov Process Based Method Researching of Ship Maintenance Dock Demand Evaluation

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Abstract. Prediction of support resources has always been the focus and difficulty for ship comprehensive support. In this paper, a prediction model and method are proposed for the support needs of ship maintenance docks. Markov process is introduced and applied to the forecasting of ship dock demand. By analyzing the system characteristics of ship dock use, a Markov forecasting model and evaluation index of dock demand are established, and the validity of the forecasting method is verified by case analysis. The calculation example shows that the prediction model can better reflect the influence of ship maintenance intensity and the number of docks on the system state, and the optimal dock requirements reflected by the evaluation index are in good agreement with the actual engineering. The research shows that the forecasting model of ship maintenance dock demand proposed in this paper can better reflect the physical process and engineering practice, and can provide theoretical support for decision analysis, which has good practical significance.

Key words: Ship equipment; Maintenance; Dock requirement; Markov process.

1. Introduction

As the increasing emphasis on maritime rights and the development of maritime industry in various countries, the number of ships will increase by spurt in the future, so it is particularly important to plan the support resources scientifically and reasonably[1]. Dock is the main content of ship maintenance support resources and an important place to carry out ship maintenance. All planned maintenance, important emergency repairs and maintenance operations below the waterline need to be arranged in dock[2]. Planning and construction of docks need to consider factors comprehensively such as site selection, tidal level, access channels, supporting facilities, maintenance requirements, security and confidentiality, and economy. Therefore, countries are more cautious about dock construction[3~4]. Scientifically predicting the future demand for the number of docks, avoiding resource shortages and waste, is of great significance for planning the construction of support capabilities, improving ship support capabilities, and enhancing support efficiency[4].

In the field of ship equipment support resource demand assessment research, structured and semi-structured quantitative analysis methods, or comprehensive evaluation methods based on Analytic Hierarchy Process (AHP) are commonly used in China[5~11]. The disadvantage of these methods is that their evaluation models are generally based on equipment reliability information, usage information, big data statistical information, or overly rely on expert opinions, which limits the scope of engineering applications.

Markov theory is an important branch of stochastic process of modern probability theory, which is widely used in communication, control, social science and other technical fields. It has a good mathematical foundation and is still a hot topic for scholars at home and abroad[12~13]. Based on the queuing theory in operational research, Markov process method is used to analyze the probability of random events, considering the possible states and transitions of events, and providing quantitative support for scientific decision-making by establishing mathematical differential equations of state transition. The physical process of ship queuing for docking maintenance has the property of Markov process, so this paper will discuss the application of Markov process method to the quantity demand assessment of ship maintenance docks, and verify the effectiveness of the evaluation model and method by establishing mathematical modeling and case analysis, hoping to provide theoretical support for dock planning and development.

2. Physical model of docking maintenance process

When a ship is at sea, minor malfunctions are handled by the crew, and when unresolved malfunction occurs, need to return to the home port for docking and maintenance. If more ships need to return to the port for maintenance than the number of docks, there will be a situation where ships queue up for maintenance until a dock is empty. It can be seen that the ship's demand for returning to port and the strength of dock repair determine the queuing state of the ship to be repaired. During the queuing process, ships are constantly docked in and docked out, and the occupation or release state of the dock is in a dynamic change.

To establish a mathematical model for evaluating the quantity demands of dock, this paper defines a physical model for dock maintenance based on the actual process of ship maintenance, as shown in Figure 1, and refers to it as the docking usage system.

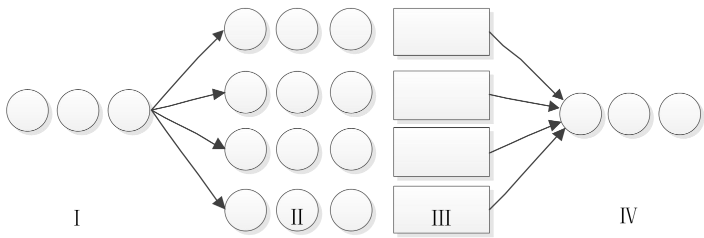

Fig. 1 Queuing model of docking usage system

As shown in Figure 1, the model consists of four elements. Sample demand (Ⅰ) refers to the ships needs to be repaired in urgent from the sea, waiting queue (Ⅱ) refers to ship waiting in line for dock maintenance, maintenance channel (Ⅲ) refers to the configured maintenance dock, and recovery after repair (IV) refers to ships leaving the dock after completing maintenance.

For example, assuming that there are *m* ships in urgent repair status (sample requirement) in the system, and *n* docks (maintenance channels), each ship may return for repair at any time. When *m*>*n*, that is, the number of ships in the system is greater than the docks, the docks are fully occupied, and some ships are in a queue waiting state until one of the ships docked out after completing maintenance.

3. Markov evaluation model of dock demands

3.1 Markov process

Markov processes mainly examine the transition patterns between different states of a system and have wide applications in many fields[13]. This article explores the application of Markov processes to evaluate the optimal number of ship maintenance docks. As mentioned earlier, Markov process is a typical stochastic process that can be expressed as a stochastic process { $X(t)$ } with continuity and state space $\psi=\{0，1,2\ldots，r\}$. Assuming that the state of the process at time *s* is $X(s)=i$, the probability of state being *j* at time *s*+*t* satisfies equation (1):

$$\begin{aligned} &P[X(s+t)=j \mid X(s)=i, X(u)=x(u), 0 \leqslant u < s] \\ &=P[X(s+t)=j \mid X(s)=i] \end{aligned} \tag{1}$$

Where { $X(u)=x(u), 0\leqslant u<s$ } represents the "historical" state of the process before time *s*, $X(u)$ represents the random event at time *u*, and $x(u)$ represents a specific state of the random event at time *u*. Equation (1) indicates that the state of the system after time *s* is independent of the state before *s*. A stochastic process with this property is called Markov property or "no aftereffect" or "no memory".

3.2 Establishment of Markov Model for Docking Usage System

Assuming that the docking usage system consists of *m* ships and *n* maintenance channels, where the number of ships in the system is greater than the number of maintenance channels ($m>n$), the system may be in one of the following states:

x_0: All ships are at sea, and the dock maintenance channel is empty;

x_k: *k* ships returned to the dock from the sea for maintenance, and *m-k* ships were at sea ($1 \leqslant k \leqslant n$);

x_n: *n* ships returned to the dock from the sea for maintenance, and all maintenance channels were occupied;

x_m: All ships return to the dock for maintenance, of which *n* ships undergoing maintenance and *n-m* ships in a queue waiting state.

Therefore, there are $m+1$ discrete states in docking usage system, and the system is in one of them. Both docking in and docking out in the system will change the state. Without considering the minimal probability event that two ships entering and exiting the same dock at the same time, the tightly behind state of the system is only related to current state but not to the tightly before state. Therefore, it is considered that the state change of docking usage system has Markov property, and the Markov state transition differential equation can be used to model and reflect the state transition law of the system[14-15].

In order to better understand the concept of system state transformation, this article first introduces a simplified model to analyze the two possible states of a ship, namely, the ship may be at sea (x_0) or at a maintenance base (x_1). The system state transformation is shown in Figure 2.

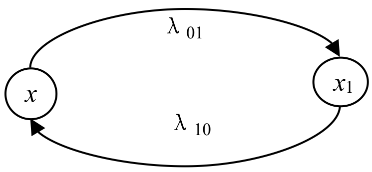

Fig. 2 Simple model of state transition

The system transitions from state x_0 to state x_1 with intensity λ_{01}, or from state x_1 to state x_0 with intensity λ_{10}. Therefore, the probability change of the system state satisfies the following equation:

$$\begin{cases} \dfrac{\mathrm{d}P_0(t)}{\mathrm{d}t} = -\lambda_{01}P_0(t) + \lambda_{10}P_1(t) \\ \dfrac{\mathrm{d}P_1(t)}{\mathrm{d}t} = -\lambda_{10}P_1(t) + \lambda_{01}P_0(t) \end{cases} \tag{2}$$

$$P_0(t=0)=1,\ P_1(t=0)=0 \tag{3}$$

$P_0(t=0)=1$, $P_1(t=0)=0$ means the state $t=0$, when there is one ship, the state probability of empty dock (P_0) is 1, so the state of one ship (P_1) in the dock is 0. The equations describe the unstable operating conditions. In order to obtain the stable operating conditions, the system equation can be taken as a derivative of time and set to zero, so the equations can be expressed by Formula (4):

$$\begin{cases} -\lambda_{01}P_0 + \lambda_{10}P_1 = 0 \\ -\lambda_{10}P_1 + \lambda_{01}P_0 = 0 \\ \sum P_i = 1 \end{cases} \tag{4}$$

By solving equation group (4), the probability of the system state can be obtained, and the results can be used to calculate and analyze evaluation indicators. Referring to the simplified model system state transition analysis method mentioned above, the state transition diagram shown in Figure 3 can be established for the docking usage system consisting of *m* ships and *n* dock maintenance channels, where *k* represents *k* ships in the system for dock maintenance.

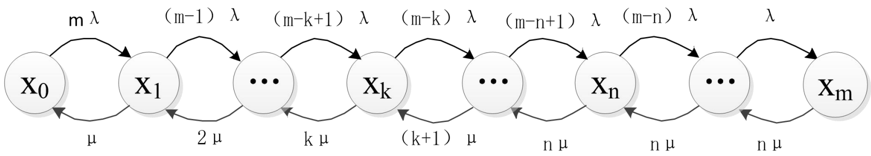

Fig. 3 Ship docking state transition diagram

Assuming there are m ships in the system, the maintenance demand intensity for each ship is constant λ, that is, λ is a constant; Each ship may be repaired in any of several maintenance channels, and the repair strength of the ship in each channel is constant μ, that is, μ is a constant. When the system transitions from one state to another, looking from left to right, the demand for a sample of ships to be repaired is proportional to the number of ships, changing from $m\lambda$ to λ; From right to left, the repair intensity is $n\mu$ before the state x_n, while between the state x_n and the state x_0, the repair intensity is proportional to the occupied channels, from $n\mu$ to μ.

In order to quantitatively describe the state of the docking usage system, the state probability P_i can be used to represent the probability that the system is in state x_i:

$$P_i = P[X(t) = x_i],\ i \in \overline{1,m} \qquad (5)$$

Referring to the analysis method given in Equation (2) and combining with the system state transition diagram, the following system state transition equation can be constructed:

$$\begin{cases} \dfrac{dP_0}{dt} = -m\lambda P_0 + \mu P_1 \cdots\cdots (6\text{-}1) \\ \dfrac{dP_k}{dt} = -[(m-k)\lambda + k\mu]P_k + (m-k+1)\lambda P_{k-1} \\ +(k+1)\mu P_{u+1}, k \in (1,2,\ldots,n-1) \cdots\cdots (6\text{-}2) \\ \dfrac{dP_n}{dt} = -[(m-n)\lambda + n\mu]P_n + (m-n+1)\lambda P_{n-1} + n\mu P_{n+1} \cdots\cdots (6\text{-}3) \\ \dfrac{dP_{n+z}}{dt} = -[(m-n-z)\lambda + n\mu]P_{n+z} + (m-n-z+1)\lambda P_{n+z-1} \\ +n\mu P_{n+z+1}, z \in 1,2,\ldots,(m-n-1) \cdots\cdots (6\text{-}4) \\ \dfrac{dP_m}{dt} = -n\mu P_m + \lambda P_{m-1} \cdots\cdots (6\text{-}5) \\ \sum_{k=0}^{n} P_u + \sum_{z=1}^{m-n} P_{n+z} = 1 \cdots\cdots (6\text{-}6) \end{cases}$$

In the equation, P_k represents the state probability that k ships need to be docked for maintenance. As shown in the state transition relationship in Figure 3. The range of k in equation (6-2) is (1, n-1); P_n represents the state probability that n ships need to be docked for maintenance, and the dock is completely occupied at this point. When more ships need to be docked for maintenance, the system state probability is P_{n+z}. For equation (6-4), P_{n+z} represents the previous state of P_m, that is, the maximum value of n+z is m-1, so the maximum value of z is m-n+1; When all ships return to port, the system status is P_m, and there is a relationship in equation (6-5).

At the initial moment, all maintenance channels in the system are empty, so the initial condition of the differential equation is:

$$P_0(t=0) = 1, P_k(t=0) = 0, k \in 1,2,\cdots,n \qquad (7)$$

The solution of the equation provides probabilities for all states.

$$P_0(t), P_1(t), \cdots P_k(t), \cdots, P_n(t)$$

Because probability is a function of time, in the analysis process, the limit value of state probability is usually used to represent:

$$P_0, P_1, \cdots, P_k, \cdots, P_n$$

The probability of all channels being free can be determined by equation (8):

$$P_0 = \left[\sum_{k=0}^{n} \frac{m!}{k!(m-k)!}\rho^k + \sum_{k=n+1}^{m} \frac{m!\rho^k}{n^{k-n} \cdot n!(m-k)!}\right]^{-1} \qquad (8)$$

Among them, $\rho = \lambda/\mu$。

When there are k ships in the system under maintenance, the probability is:

$$P_k = \frac{m!}{k!(m-k)!}\rho^k P_0 \quad 1 \leqslant k \leqslant n \qquad (9)$$

3.3 Evaluating indexes

In order to quantitatively analyze the process characteristics of the docking usage system, the following evaluation indexes are defined:

Average number of ships waiting for maintenance: $M_0 = \sum_{k=m+1}^{m} (k-n)P_k$;

Ship waiting service coefficient: $K_{np} = \frac{M_0}{m}$;

Average number of ships under maintenance and waiting for maintenance: $M_{o\delta} = \sum_{k=1}^{m} kP_k$;

Average number of occupied maintenance channels: $N_3 = \sum_{k=1}^{n} kP_k + \sum_{k=n+1}^{m} (k-n)P_k$;

Maintenance channel occupancy coefficient: $K_3 = \frac{N_3}{n}$;

Average number of free maintenance channels: $N_{cb} = \sum_{k=1}^{n} (n-k)P_k$;

Free maintenance channel coefficient: $K_{np}^{'} = \frac{N_{cb}}{n}$ 。

Combined with the system state probability, according to the following analysis process, the ship dock indexes can be calculated and analyzed. The optimal number of docks can be reflected through the evaluation index, which provides quantitative support for scientific decision-making.

Firstly, based on the usage of the number of ships by the user, combined with the estimation of the docking maintenance and repair intensity of the equipment, the number n of ships, the maintenance demand intensity λ and the repair intensity μ are given, and the number n of series docks is driven;

Then, a Markov prediction analysis model is established to systematically model and calculate the number of a series of continuous dock quantities.

Finally, the characteristics of evaluation indexes under different dock numbers are evaluated to provide quantitative support for decision analysis.

4. Case analysis

In the optimal task decision, the optimal solution is selected by changing the number of maintenance channels. Assuming that there are 8 ships in the home port that require maintenance support, the number of ship docks for maintenance is calculated as n=1, 2, 3 and 4 respectively. According to the statistics of the time interval between two shipyard repairs for each ship, assuming three scenarios: 200 days, 120 days, and 90 days, the intensity of ship docking maintenance in unit time is λ_1=0.005, λ_2=0.008 and λ_3=0.011； Assuming that the maintenance time for a single shipyard is 45 days, that is, the repair strength of the dock per unit time is μ=0.022.

Taking the docking usage system (m=8, n=1) as an example, the system state transition equations can be expressed as:

$$\begin{cases} -8\lambda P_0 + \mu P_1 = 0 \\ -(7\lambda+\mu)P_1 + \mu P_2 + 8\lambda P_0 = 0 \\ -(6\lambda+\mu)P_2 + \mu P_3 + 7\lambda P_1 = 0 \\ -(5\lambda+\mu)P_3 + \mu P_4 + 6\lambda P_2 = 0 \\ -(4\lambda+\mu)P_4 + \mu P_5 + 5\lambda P_3 = 0 \\ -(3\lambda+\mu)P_5 + \mu P_6 + 4\lambda P_4 = 0 \\ -(2\lambda+\mu)P_6 + \mu P_7 + 3\lambda P_5 = 0 \\ -(\lambda+\mu)P_7 + \mu P_8 + 2\lambda P_6 = 0 \\ -\mu P_8 + \lambda P_7 = 0 \\ \sum_{j=1}^{8} P_j = 1 \end{cases} \quad (10)$$

Substituting parameters λ_1, λ_2, λ_3 and μ respectively, the probability P_i is calculated by Matlab program. Similarly, assuming that n=2,3,4, the corresponding system state equations can be solved to obtain the probability distribution curves for different ship maintenance demand intensities (λ) and different number of docks (n) under different discrete state probabilities, as shown in Fig.4 to Fig.6.

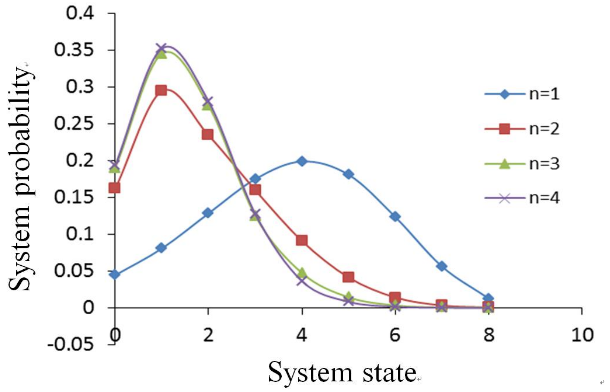

Fig. 4 Docking usage system state probability (λ=0.005)

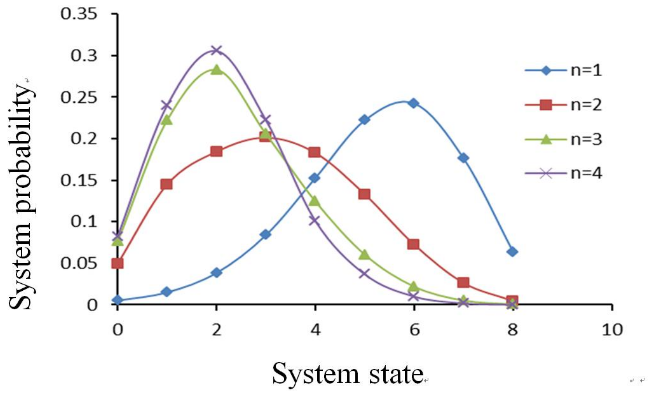

Fig. 5 Docking usage system state probability (λ=0.008)

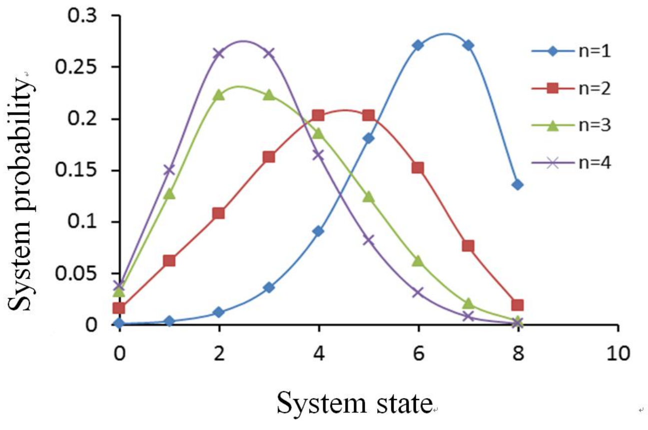

Fig. 6 Docking usage system state probability (λ=0.011)

From Fig.4 to Fig.6, it can be seen that the probability of the system state with a large number and a small number of ships queuing for maintenance in the dock is relatively small, and the maximum probability of the system depends on the number of docks and ships. The more docks there are, the smaller the number of ships to be repaired corresponding to the maximum probability state of the system, which reflects the fact that increasing the number of docks reduces the probability of ships queuing for maintenance, which is consistent with the actual use of docks. This indicates that the Markov dock demand prediction model has good applicability.

According to the evaluation indexes mentioned above, the docking usage system can be quantitatively analyzed, and the results are shown in Table 1.

Table 1. Docking usage system evaluation indexes

n	λ	Indexes					
		M_0	K_{np}	$M_{0\delta}$	K_3	N_{cb}	$K_{np}^{'}$
1	0.005	2.84	0.355	3.795	2.920	0.044	0.044
	0.008	4.27	0.534	5.264	4.285	0.005	0.005
	0.011	5.003	0.625	6.002	5.006	0.001	0.001
2	0.005	0.539	0.067	1.921	0.982	0.751	0.375
	0.008	1.417	0.177	3.172	1.312	0.340	0.170
	0.011	2.278	0.285	4.185	1.627	0.139	0.070
3	0.005	0.088	0.011	1.553	0.452	1.704	0.568
	0.008	0.336	0.042	2.380	0.580	1.369	0.456
	0.011	0.719	0.090	3.146	0.641	0.955	0.318
4	0.005	0.012	0.001	1.491	0.363	2.249	0.562
	0.008	0.063	0.008	2.18	0.496	2.368	0.592
	0.011	0.171	0.021	2.781	0.573	2.179	0.545

By analyzing the average number of ships to be repaired, it can be seen that the comparison of evaluation indexes is shown in Fig.7 under the conditions of different maintenance docks and ships to be repaired.

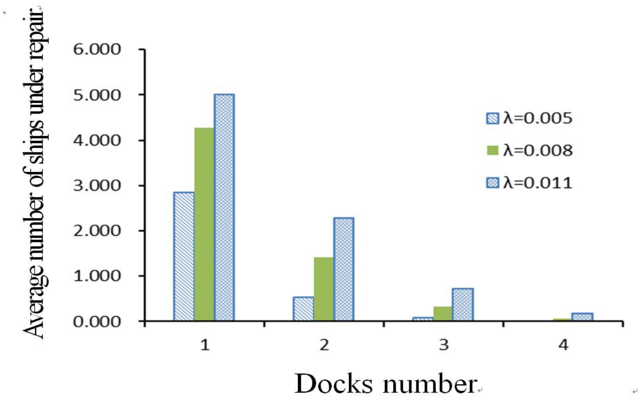

Fig. 7 Mean value of docking queue state ships

It can be seen from Fig.7 that when the number of docks is 3, the average number of ships waiting in line for repair is less than 1, which has the least impact on the ship's task execution and does not cause a lot of waste of support resources. It can be considered that the optimal number of maintenance docks is 3. Similarly, other evaluation indexes can be analyzed and evaluated, and similar analytical conclusions can be drawn.. Based on engineering experience, it is considered that the predicted results of this article are consistent with the engineering practice, which can provide effective theoretical support for decision-making.

5. Conclusion

In this paper, the characteristics of Markov process are applied to the evaluation of the optimal number of ship docks. By analyzing the maintenance process of ship docks and applying the mathematical properties of Markov process, a evaluation model and evaluation index system of ship docks are established. The case study shows that the conclusion of the mathematical model established in this paper is in good agreement with the actual situation, which has certain guiding value for the decision-making analysis of dock facilities construction planning.

Due to the limited research on the Markov process method in the field of comprehensive ship support in China, the understanding and application research of this method in this article is still shallow. For example, the model lacks of consideration for the differences between planned maintenance and emergency repair, and the evaluation criteria are not perfect enough. It is necessary to further study the evaluation model or evaluation indexes. It is expected that this study can serve as a catalyst for further exploration by researchers in this field, enrich the mathematical theory of ship equipment technical support in China, and better guide engineering practice.

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