Total stress model of clay considering small strain characteristics

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Abstract. Saturated soft clay has the characteristics of high compressibility, low strength, and high structural sensitivity. Its small strain characteristics have a significant impact on the deformation development and long-term bearing capacity of ocean engineering foundations. A total stress model for undrained cyclic loading of clay considering small strain characteristics was created based on the Overlay model. The proposed model was used to simulate the static or dynamic triaxial test results of over consolidated Chicago clay, remoulded normally consolidated Georgia kaolin, and normally consolidated clay Malaysia kaolin, respectively. The results verified that the model can simulate the static shear characteristics of clay. The model can also simulate the weakening of soil strength and strain accumulation under cyclic loading, and is suitable for analyzing the response of ocean engineering foundations under long-term cyclic loading.

Keywords: soft clay; total stress model; weakening; strain accumulation.

1. Introduction

Deep saturated soft clay is widely distributed in the eastern coastal areas of China, which has the characteristics of high natural moisture content, high compressibility, low strength, and high structural sensitivity. The small strain characteristics and cyclic weakening characteristics are the typical mechanical properties of saturated soft clay,which have a significant impact on the deformation development and long-term bearing capacity of ocean engineering foundations..

At present, there are many studies on the constitutive relationship of saturated clay based on the effective stress elastic-plastic form, mostly using critical state theory and boundary interface theory (Zienkiewicz et al., 2015; Kimoto, et al., 2015; Cheng and Yang, 2019) to describe the accumulation and dissipation of pore pressure in soil from the perspective of effective stress, and to reveal the internal mechanism of soil deformation and failure. Although these models have relatively complete theories, their formulas are often complex due to the inclusion of multiple surface movements and multiple hardening rules, resulting in significant limitations in practical engineering applications. There are relatively few constitutive studies on the total stress model, mostly based on motion hardening theory and using the Mises yield function. Total stress model has relatively simple formulas, fewer model parameters, and is easy to obtain, making it highly applicable in practical engineering.

2. Total stress model of clay considering small strain characteristics

This study constructs a total stress model for undrained cyclic loading of clay considering small strain characteristics based on nonlinear motion hardening theory and Overlay model (Benz et al., 2009). At the same time, the cyclic weakening effect of soil strength and stiffness is introduced to modify the soil motion hardening rule. This model considers the stress history effect by introducing the historical shear strain γ_{HST}. By modifying the Hardin-Drnevich formula, the relationship between historical shear strain γ_{HST} and soil shear modulus is established, and the weakening law of soil shear modulus with the development of soil shear strain is described:

$$G_{\mathrm{sec}}=\frac{G_0}{1+a\gamma_{\mathrm{HST}}/\gamma_{0.7}} \qquad (1)$$

in which, G_{sec} is the tangent shear modulus, G_0 is the initial shear modulus of the soil at a very small strain stage (<10^{-6}), $\gamma_{0.7}$ is the shear strain corresponding to the weakening of the shear modulus to 70 percent of G_0, and a is a constant that affects the shape of the curve, and it is recommended to take a value of 0.385.

Under small deformation conditions, the total strain increment $d\varepsilon_{ij}$ follows the superposition principle:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \quad (2)$$

in which, $d\varepsilon_{ij}^{e}$ and $d\varepsilon_{ij}^{p}$ represent elastic and plastic strain increments, respectively.

The yield surface of this model is the Mises yield surface, which is expressed as:

$$f = \sqrt{\frac{3}{2}}\left\|S_{ij} - \alpha_{ij}\right\| - A = 0 \quad (3)$$

in which, S_{ij} is the deviatoric stress tensor, α_{ij} is the internal variable controlling the translation of the yield surface (i.e. motion hardening), and parameter A characterizes the size of the yield surface.

Based on the correlation flow rule, the plastic strain increment can be obtained by the following equation:

$$d\varepsilon_{ij}^{p} = d\lambda\frac{\partial f}{\partial\sigma_{ij}} = d\lambda\frac{\sqrt{3/2}(S_{ij} - \alpha_{ij})}{\| S_{ij} - \alpha_{ij}\|} \quad (4)$$

in which, $d\lambda$ is the plastic multiplier, which can be derived based on the consistency condition of the yield surface:

$$d\lambda = \frac{1}{K_p}\frac{\partial f}{\partial\sigma_{ij}}d\sigma_{ij} \quad (5)$$

in which, K_p is the plastic modulus.

In the basic nonlinear motion hardening model, the Armstrong-Frederick (AF) motion hardening rule (Armstrong and Frederick, 1966) is adopted to control the translation of the yield surface:

$$d\alpha_{ij} = d\lambda\bar{\alpha}_{ij} = d\lambda\left(C\frac{\partial f}{\partial\sigma_{ij}} - \gamma\alpha_{ij}\right) \quad (6)$$

in which, C and γ are material parameters.

Based on the motion hardening rule, the plastic modulus K_p can be derived as follows:

$$K_p = -\frac{\partial f}{\partial\alpha_{ij}}\bar{\alpha}_{ij} = \frac{3}{2}C - \gamma\frac{\partial f}{\partial\sigma_{ij}}\alpha_{ij} \quad (7)$$

By establishing an association between the strength surface of the model and the undrained shear strength s_u of saturated clay under triaxial conditions, the relationship between the triaxial compressive strength s_u^{TC} and tensile strength s_u^{TE} of transversely isotropic clay and model parameters A, C, and γ can be obtained as follows:

$$s_u^{TC} = s_u^{TE} = \frac{1}{2}\left(A + C/\gamma\right) \quad (8)$$

In this way, model parameters can be calibrated based on triaxial test results.

By introducing material parameters ρ_e and ρ_s to reduce the plastic modulus K_p of saturated soft clay under cyclic loading, the formula can be correspondingly modified as follows:

$$K_p = \rho_e\left[\frac{3}{2}\rho_s C - \gamma\frac{\partial f}{\partial\sigma_{ij}}\alpha_{ij}\right] \quad (9)$$

According to equation (9), it can be seen that the stiffness weakening parameter ρ_e will reduce the plastic modulus and overall stiffness of the soil, while the strength weakening parameter ρ_s mainly reduces the soil parameter C, thereby directly affecting the size of the strength surface.

Based on present research, the weakening of soil stiffness and strength can be related to the cumulative plastic deformation of the soil, and it can be assumed that the weakening coefficient of soil stiffness ρ_e follows the following law:

$$\rho_e = \frac{1}{1 + c_d \varepsilon_d^p} \tag{10}$$

in which, ε_d^p is the cumulative plastic strain of the soil, c_d is the material parameter, and the influence of the soil stiffness weakening parameter gradually decreases with the increase of cumulative plastic strain.

The weakening coefficient ρ_s can be derived based on the weakening relationship proposed by Einav and Randolph (2010):

$$\frac{s_u}{s_{u0}} = \delta_{rem} + \left(1 - \delta_{rem}\right)\exp\left(-\frac{3\varepsilon_d^p}{\varepsilon_{95}}\right) \tag{11}$$

in which, s_u and s_{u0} represent the weakened and initial undrained shear strength, δ_{rem} equals s_u / s_{u0} in the fully remolded state, ε_{95} is the material parameter. In order to apply the formula (11) to the model in this study, it can be obtained that:

$$\frac{s_u}{s_{u0}} = \frac{A + \rho_s C / \gamma}{A + C / \gamma} \tag{12}$$

Thus, the expression for the strength weakening parameter can be derived as follows:

$$\rho_s = \frac{s_u}{s_{u0}} + \left(\frac{s_u}{s_{u0}} - 1\right) A \frac{\gamma}{C} \tag{13}$$

3. Test simulation and constitutive model verification

This study simulates the monotonic triaxial shear tests or cyclic triaxial tests of naturally light overconsolidated clay Chicago clay (Finno and Kim, 2016), remoulded normally consolidated clay Georgia kaolin (Sheu, 1985), and normally consolidated clay Malaysia kaolin (Yu et al., 2000; Ho, 2013) to verify the prososed model.

The tiaxial tests of Chichago clay (Finno and Kim, 2016) are simulated to verify the influence of stress history on the small strain stiffness of soil using the proposed model. Two soil samples with different overburden depths (referred to as "deep soil" and "shallow soil") were selected for comparison of undrained triaxial compression (CKcUTC) and tensile (CKcUTE) tests. The parameters are shown in Table 1.

Table 1 Parameters of Chicago clay in simulation

Parameter	E/Mpa	v	A/kPa	C/Mpa	γ	G_0/Mpa	$\gamma_{0.7}$	c_d	δ_{rem}	ε_{95}
deep soil	38.1	0.49	12	45.7	480	100	2×10^{-4}	0	1	/
shallow soil	26.7	0.49	8	36	480	85	2×10^{-4}	0	1	/

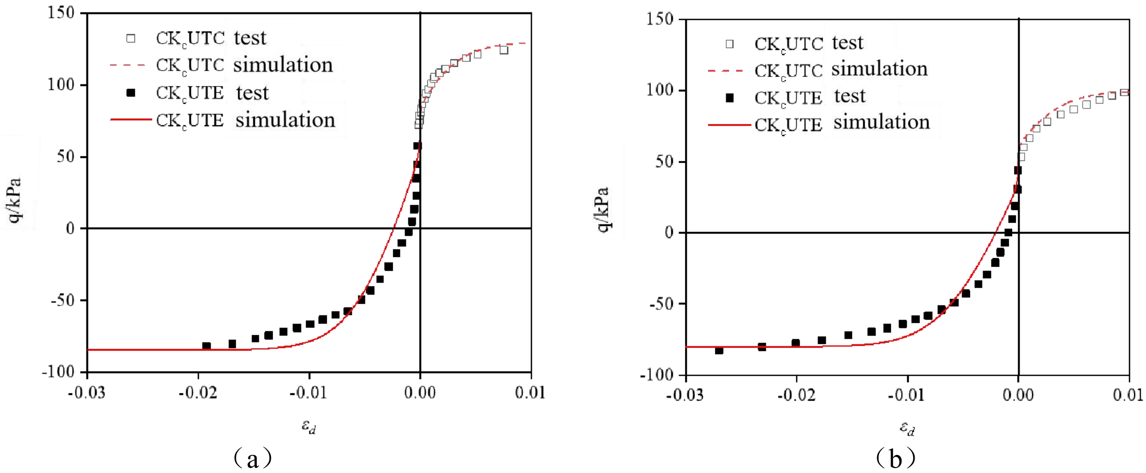


Figure 1 Comparison of CKcUTC and CKcUTE test and simulation results of Chicago clay: (a) deep soil; (b) shallow soil

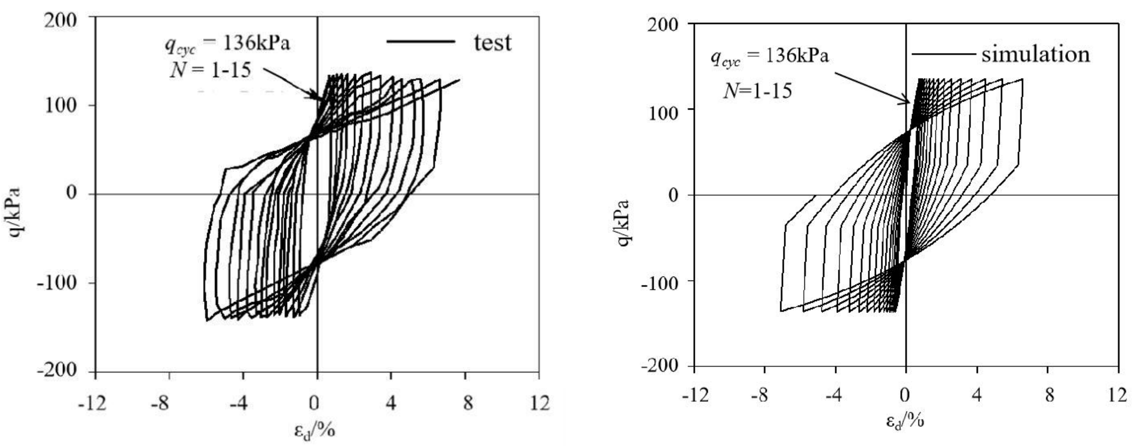
Figure 1 shows a comparison between the test and simulation results of consolidated undrained triaxial compression and tension of Chicago clay in deep and shallow layers. The simulation results indicate that the total stress model can well reflect the nonlinear stress-strain relationship of soil under small and large strains, as well as the undrained shear strength of soil.

This study selects stress bidirectional cyclic loading tests of Georgia kaolin (Sheu, 1985) and strain controlled cyclic loading tests of Malaysia kaolin (Yu et al., 2000; Ho, 2013) to verify the proposed model. The soil parameters used in simulations are shown in Table 2.

Figure 2 compares the stress controlled cyclic triaxial test results and the simulation results of Georgia kaolin. The test conducted a cyclic loading test with a stress amplitude of 136kPa. Due to the limited number of cycles and the insignificant weakening of soil strength, the parameter δ_{rem} value was 1.0 (i.e. ignoring the weakening of soil strength). It can be seen that the model can reasonably simulate that the stiffness of the soil gradually decreases with the increase of the number of cycles, and the cumulative strain of the cycles is in good agreement with the test.

Table 2 Parameters for simulating bidirectional cyclic loading tests

Parameter	p_c'/kpa	K0	E/Mpa	v	A/kPa	C/Mpa	γ	G0/Mpa	γ 0.7	cd	δ_{rem}	ε_{95}
Georgia kaolin	345	1	33.8	0.49	50	33.8	240	51.7	1×10^{-4}	0.04	1	/
Malaysia kaolin	100	1	11.2	0.49	10	16	350	62.2	1×10^{-4}	0.02	0.33	1.89



（a） （b）

Figure 2 Cyclic triaxial test results of Georgia clay: (a) test results; (b) simulation results

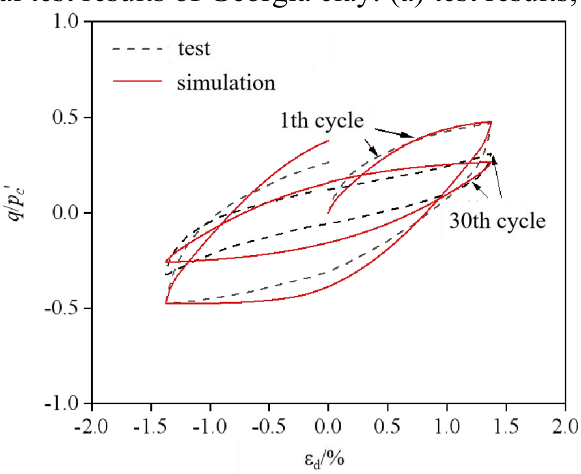


Figure 3 Cyclic triaxial test and simulation results of Malaysian clay

Figure 3 shows the test and simulation results of strain controlled bidirectional cycling for 30 times under an average effective confining pressure of 100kPa of Malaysia clay. The cyclic weakening parameters of soil can be fitted based on the T-bar test (Yu et al., 2018). The simulation results show that the proposed model can reasonably reflect the trend of soil shear strength gradually decreasing with the increase of cyclic loading times, and it is in good agreement with the test results when the number of cycles is small. When the number of cycles is large, the simulation results overestimate the shear strength of the soil to a certain extent. Overall, the simulation results demonstrate that the proposed model can reasonably simulate the cyclic weakening effect of soil, and has a high degree of agreement with the test results.

4. Summary

This study describes the stress-strain relationship of undrained clay under cyclic loading based on a nonlinear motion hardening model, and introduces an Overlay small strain model to consider the influence of stress paths on the small strain stiffness of soil. Weakening parameters were introduced to construct a total stress cyclic loading constitutive model of undrained clay that can consider the small strain characteristics and cyclic weakening characteristics of soil. By comparing the simulation results with existing monotonic and cyclic loading triaxial tests, it was verified that the proposed model can simulate the static shear characteristics of soil under monotonic loading and has reasonable simulation results for the weakening of soil strength under cyclic loading and the accumulation of soil strain. The proposed model is suitable for analyzing the response of ocean engineering foundations under long-term cyclic loading.

Acknowledgements

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