MVDR Algorithm Based on Loading Factor and Sample Selection Strategy

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Abstract. The MVDR (Minimum Variance Distortionless Response) algorithm is a classic Wiener filtering method used for beamforming in array signal processing. In a one-dimensional linear array high-frequency ground wave radar system, it can be employed to suppress various types of ionospheric clutter. A key step in suppression is the use of maximum likelihood estimation (MLE) to estimate the ionospheric clutter covariance matrix. However, MLE typically assumes that samples are independently and identically distributed (i.i.d.). Traditional MVDR algorithms estimate the clutter covariance matrix using all samples, which may not satisfy the i.i.d. condition. Therefore, this paper proposes a new sample selection strategy that utilizes the Mahalanobis distance method to select samples. Meanwhile, due to the very small numerical values of the ionospheric clutter covariance matrix, even if the matrix is full-rank, numerical instability may occur during inversion. To address this issue, the paper introduces 4 different forms of regularization factors. Empirical data demonstrate that the proposed new method offers better suppression of ionospheric clutter compared to the traditional MVDR algorithm.

Keywords: MVDR; loading factor; Mahalanobis distance.

1. Introduction

High-Frequency Surface Wave Radar is primarily used for detecting ground targets and monitoring the ocean surface[1]. It utilizes high-frequency signals that propagate along the Earth's surface and can penetrate through terrain obstacles. Ionospheric clutter is a major source of interference for HF ground wave radar, primarily arising from the irregularities in the ionosphere and the propagation characteristics of electromagnetic waves within the ionosphere[2].

To suppress the ionospheric clutter, many researchers have conducted extensive studies. Reference [3] proposed using the oblique projection method to suppress ionospheric clutter. Its advantage lies in its simplicity, requiring only a single sample and not necessitating the estimation of the clutter covariance matrix. However, it has the drawback of requiring a high signal-to-noise ratio. Reference [4] proposed using the GSC (Generalized Sidelobe Canceller) algorithm to suppress ionospheric clutter. The advantage of this method is that the algorithm is relatively mature, hardware implementation is simple, and it can effectively reduce the impact of ionospheric clutter, thereby improving the signal-to-noise ratio of the target signal. However, its drawback is that it requires a high-quality blocking matrix, and the effectiveness of the suppression is entirely dependent on the blocking matrix[5,6].

This paper first uses the traditional MVDR algorithm to suppress the ionospheric clutter. In this process, the estimation of the clutter covariance matrix is crucial. Since the total clutter samples are not independent and identically distributed, the estimated clutter covariance matrix differs significantly from the ideal clutter covariance matrix, resulting in poor suppression of ionospheric clutter. Therefore, this paper proposes a new sample selection strategy. The role and purpose of pattern recognition is to correctly classify a specific piece of information (pattern) into a category; its core is to design an appropriate classifier, i.e., classification criteria. The Mahalanobis distance method can serve as the classification criterion for sample selection. In the High-Frequency surface wave radar systems, the numerical instability of the ionospheric clutter covariance matrix is a complex challenge. To address this issue, this paper introduces four different forms of regularization factors. Simulations with measured data show that, compared to the traditional MVDR algorithm, the new algorithm proposed in this paper provides better suppression of the ionospheric clutter.

2. Experimental Principle

2.1 MVDR Algorithm

The MVDR algorithm is a beamforming technique primarily used in array signal processing. Its purpose is to maximize the reception of signals from a specific direction while minimizing interference and noise from other directions. This algorithm finds wide application in fields such as radar, sonar, and wireless communications[7,8].

The specific steps of the MVDR algorithm are as follows:

1) Define the Array Signal Model: Assume the array has N sensors, the received signal can be represented as:

$$x = s + c + n \tag{1}$$

where s is the target signal and c is the ionospheric clutter and n is the noise .

2) Construct the Array Steering Vector: For a signal coming from direction θ, its array steering vector is $a(\theta)$.

3) Calculate the Covariance Matrix: Estimate the covariance matrix R of the received signals based on the data:

$$R = x(n)x^H(n) \tag{2}$$

where $x^H(n)$ is the conjugate transpose of $x(n)$.In practical applications, the clutter covariance matrix usually cannot be obtained directly but is instead estimated from neighboring clutter cells.

$$R = \frac{1}{K}\sum_{i=1}^{K} x_i(n){x_i}^H(n) \tag{3}$$

4) Formulate the Optimization Problem: The goal of the MVDR algorithm is to find a weight vector W that minimizes the output power while maintaining a distortionless response in the target direction. The optimization problem can be expressed as:

$$\min_W W^H RW \quad subject\,to \quad W^H a(\theta) = 1 \tag{4}$$

5) Solve for the Weight Vector: Using the method of Lagrange multipliers, the optimal weight vector W can be obtained as:

$$W = \frac{R^{-1}a(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{5}$$

6) Form the Beamformed Output: Apply the optimal weight vector W to the received signal to obtain the beamformed output:

$$y(n) = W^H x(n) \tag{6}$$

To summarize, the steps of the MVDR algorithm include defining the array signal model, constructing the array steering vector, calculating the covariance matrix, formulating the optimization problem, solving for the weight vector, and finally forming the beamformed output.

2.2 Mahalanobis Distance

Mahalanobis distance is a type of similarity measure. A similarity measure is a specific metric used to express the degree of similarity between samples, where similar samples are grouped into a cluster. There are various types of data and numerous similarity measures. This section primarily introduces the Mahalanobis distance[9,10].

In general, samples of the same class have similar features, while samples of different classes exhibit significantly different features. This means that samples of the same class cluster in one region, whereas samples of different classes are relatively distant from each other. The distance between sample points in the feature space directly reflects the category to which the corresponding samples belong and can be used as a measure of sample similarity. The closer the distance, the greater the similarity, and the higher the likelihood that they belong to the same class; conversely, the farther the distance, the smaller the similarity, and the lower the likelihood that they belong to the same class. Therefore, distance measures are also referred to as dissimilarity measures, which can be converted into similarity measures by methods such as taking the negative value.

The Mahalanobis distance for a complex signal is given by:

$$d(x_i, x_j) = \sqrt{(x_i - x_j)^H R^{-1} (x_i - x_j)} \tag{7}$$

2.3 Loading Factor.

The MVDR algorithm is primarily designed for narrowband signals and operates based on interference suppression at a specific frequency[11]. However, ionospheric clutter is typically a broadband signal that contains multiple frequency components. When handling broadband signals, the MVDR algorithm needs to be applied individually at each frequency point[12]. Since ionospheric clutter is non-stationary in time, the clutter covariance matrix exhibits numerical instability. This paper presents a solution by introducing a loading factor. The loading factor consists of two parts: the magnitude λ and the form Φ of the matrix. The discussion on the magnitude will be carried out later; this section first discusses the form of the matrix. The paper proposes four forms: an all-ones matrix, an identity matrix, a uniformly distributed matrix, and a normally distributed matrix. The advantages and disadvantages of these four forms are thoroughly discussed in the simulation results.

$$\hat{R} = R + \lambda \Phi \tag{8}$$

3. Simulation Results

We first analyze and study a batch of high-frequency surface wave radar echo data containing ionospheric clutter. This data was obtained from the Weihai radar station in July 2015. The radar operated at a frequency of 5.705 MHz, with the receiving array consisting of 8 elements. Each CPI (Coherent Processing Interval) contained 4480 pulses, and Doppler processing was performed on the data to form 4480 Doppler bins. There were 300 range bins, with each range bin corresponding to 1.5 km.The range-Doppler spectrum of the data is shown in the figure below.

It can be observed that between 90 km and 450 km, there is a significant amount of ionospheric clutter with various shapes. This clutter occupies a large number of range-Doppler cells, severely affecting target detection.

From the figure, it can be observed that the 67th, 120th, and 150th range bins are all covered with a significant amount of ionospheric clutter. The average clutter power for each of these rangebins is calculated to be -111.1785 dB, -128.8049 dB, and -124.9030 dB, respectively.

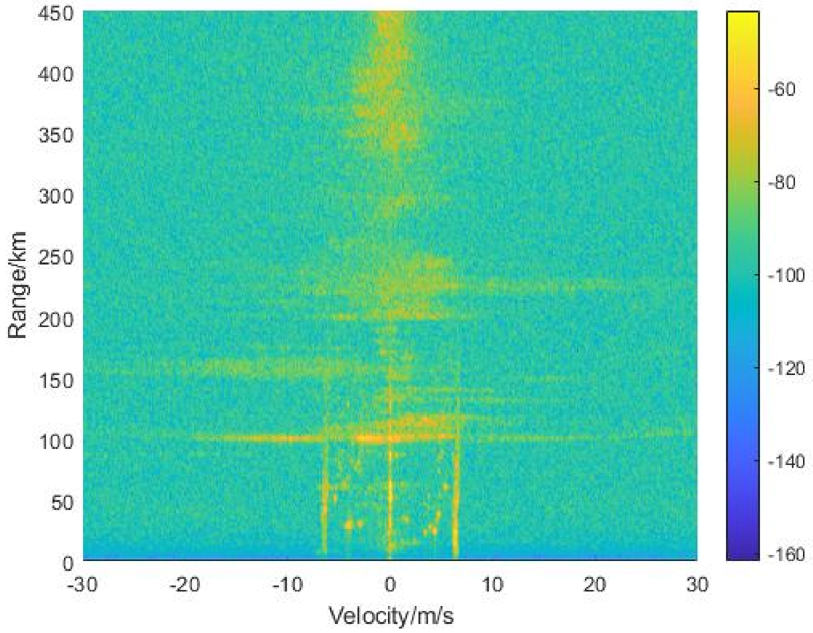

Fig. 1 Range-Doppler spectrum of ionospheric clutter

To verify the effectiveness of the proposed method for detecting targets in the presence of ionospheric clutter, a target was added to this batch of data. The target has an average power of -105 dB and a signal-to-clutter ratio of 6.1785 dB. The direction of the target is 5 degrees, and the target's velocity is -2.8 m/s.

After adding the target, Doppler processing and digital beamforming were performed. The resulting processed data is shown in the figure below.

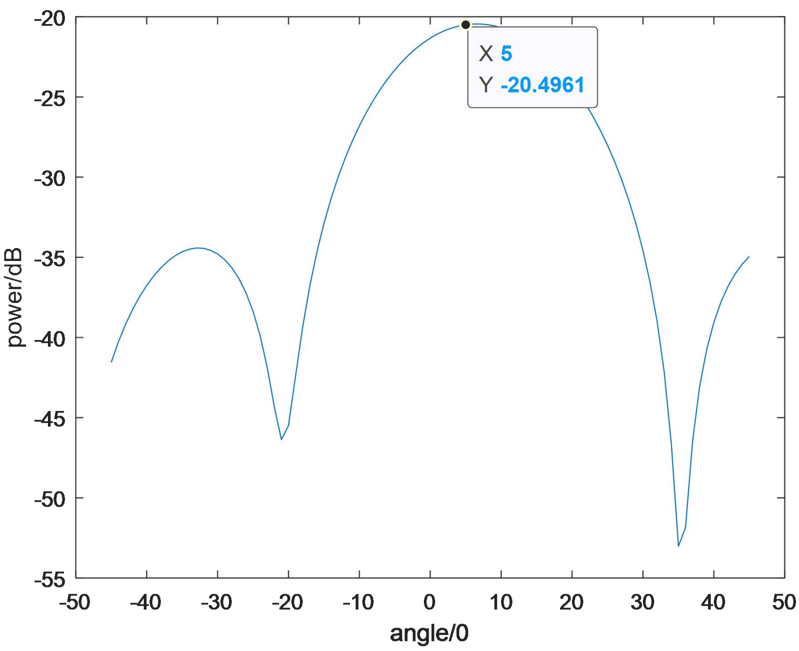

Fig. 2 Digital beamforming

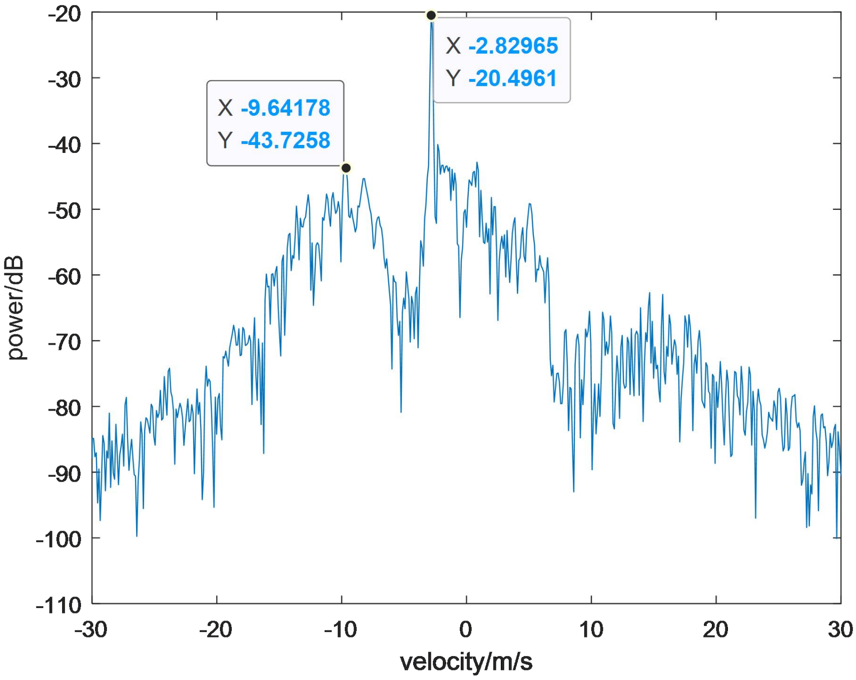

Fig. 3 Doppler processing

The results obtained by applying the traditional MVDR algorithm to the measured data are shown in the figure below. As can be seen from the figure, the traditional MVDR algorithm provides a certain level of suppression for ionospheric clutter.

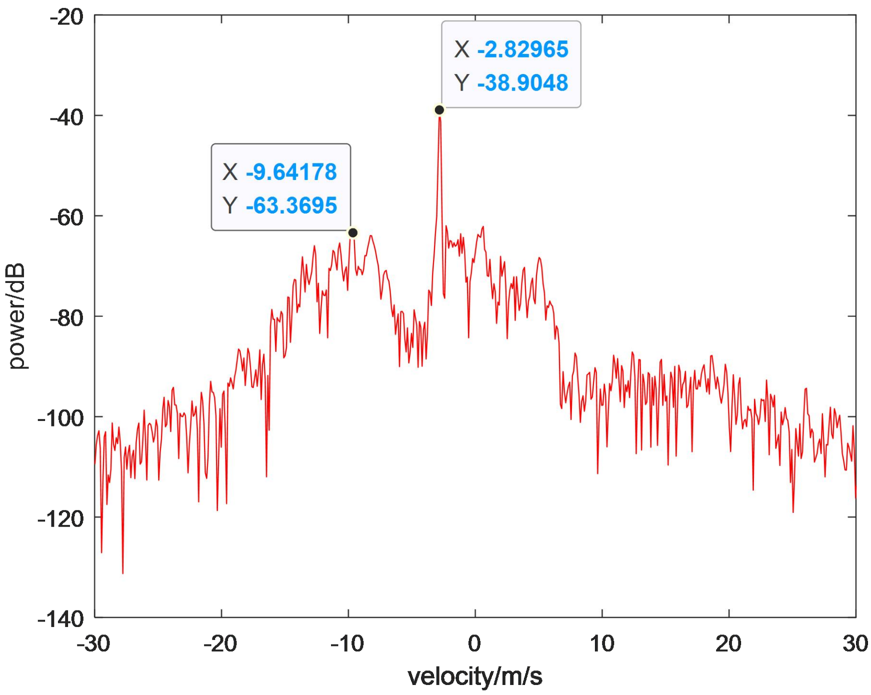

Fig. 4 MVDR processing

The MVDR algorithm uses the maximum likelihood estimation method to estimate the ionospheric clutter covariance matrix. However, the maximum likelihood estimation method requires all samples to be independent and identically distributed (i.i.d.), which is clearly not the case for ionospheric clutter samples. Therefore, this paper employs the Mahalanobis distance method to select samples that best approximate the i.i.d. condition. The 67th range bin was chosen as the test sample, and the Mahalanobis distances between this sample and all other range bins were measured. The simulation results are shown in the figure.

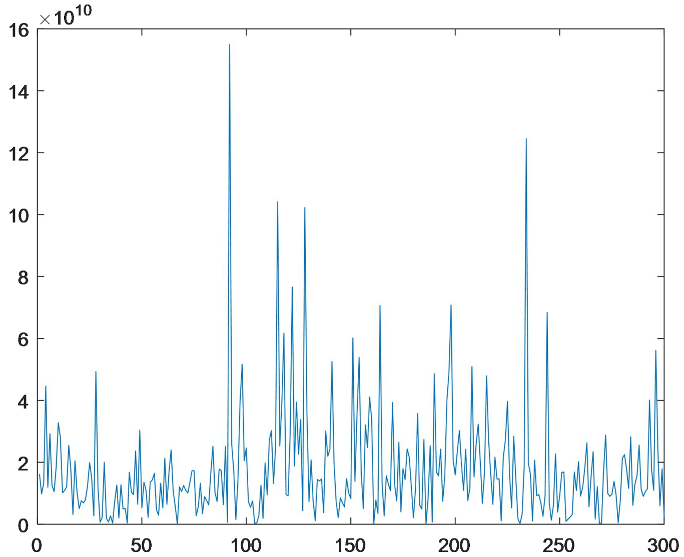

Fig. 5 Mahalanobis distance

In general, similar samples have comparable features, and the closer the distance, the greater the similarity. Therefore, a threshold can be set such that when the Mahalanobis distance is below this threshold, the range bins are considered to have ionospheric clutter that approximates the independent and identically distributed (i.i.d.) condition with the 67th range bin. These bins are then selected as training samples to estimate the ionospheric clutter covariance matrix. The simulation results are shown in the figure.

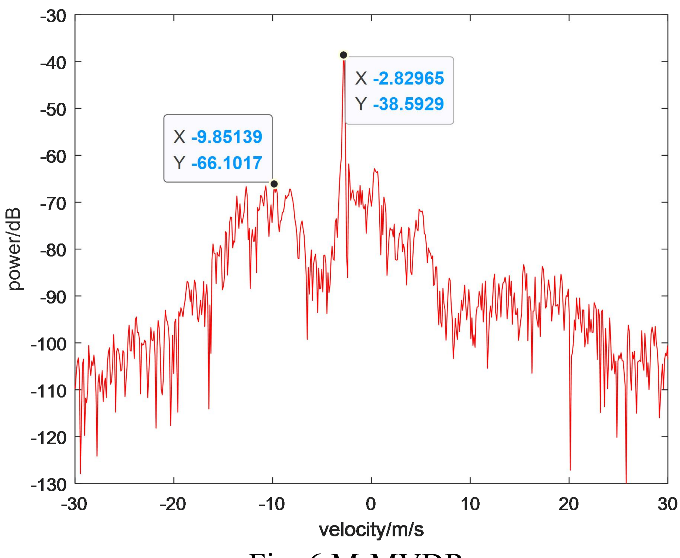

Fig. 6 M-MVDR

Since ionospheric clutter is non-stationary in time, the clutter covariance matrix exhibits numerical instability. This paper presents a solution by introducing a loading factor. The loading factor consists of two parts: the magnitude λ and the form Φ of the matrix.Measured data indicate that the magnitude λ of the matrix should be smaller than the magnitude of the clutter covariance matrix by three orders of magnitude to achieve the best processing results.When the form Φ of the matrix is normally distributed matrix, the processing results are shown in the figure.

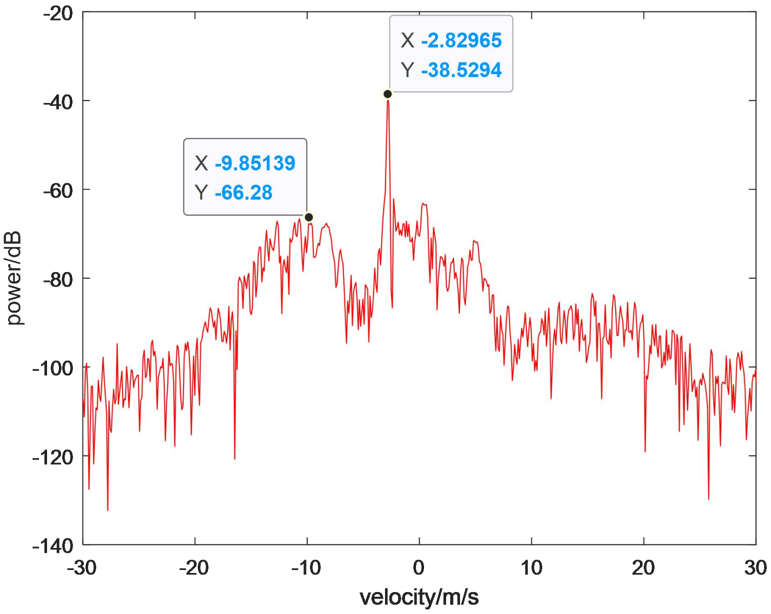

Fig. 7 N-M-MVDR

When the form Φ of the matrix is uniformly distributed matrix, the processing results are shown in the figure.

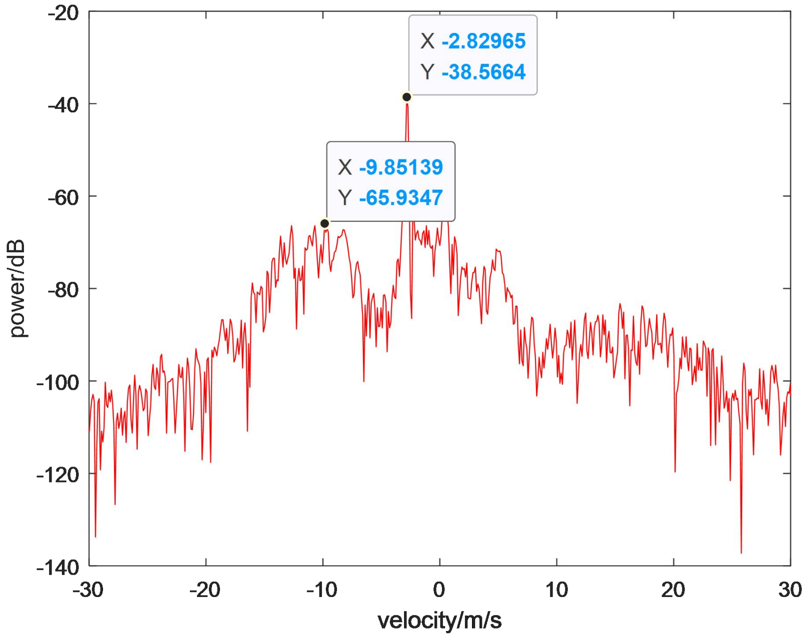

Fig. 8 U-M-MVDR

When the form Φ of the matrix is identity matrix, the processing results are shown in the figure.

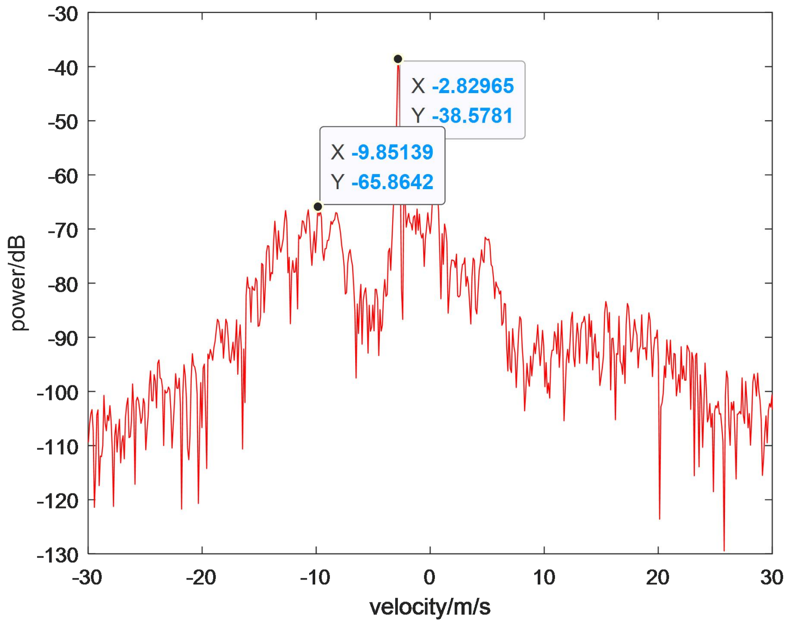

Fig. 9 I-M-MVDR

When the form Φ of the matrix is all-ones matrix, the processing results are shown in the figure.

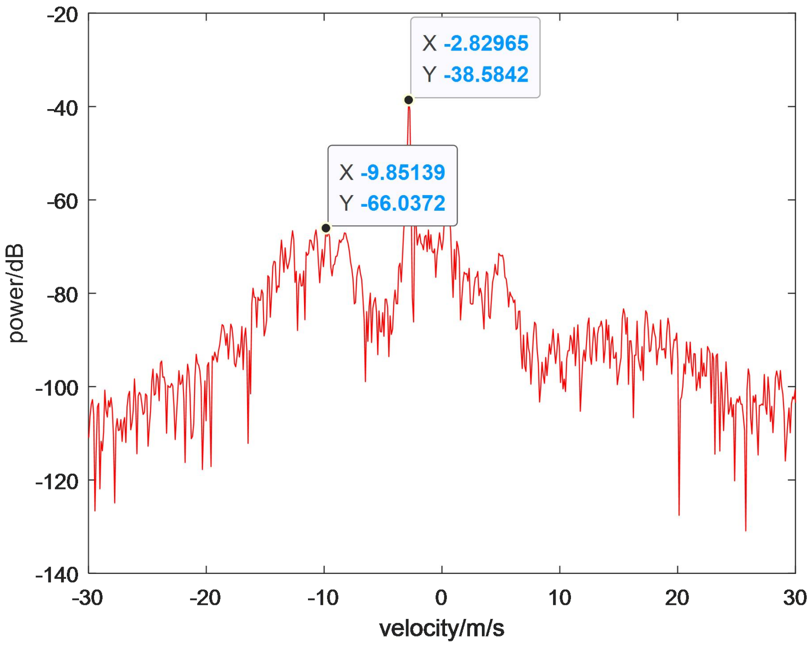

Fig. 10 A-M-MVDR

Based on the previous text, the initial SCR(signal-to-clutter ratio) is 6.1785 dB. To facilitate comparison of the effectiveness of various algorithms in suppressing ionospheric clutter, a table has been specially prepared. The units of the signal-to-clutter ratio in the table are all in dB.From Table 1, it can be seen that various algorithms can effectively improve the SCR. However, the N-M-MVDR algorithm has the best performance in suppressing ionospheric clutter, achieving an SCR of 27.7199 dB. This is because the statistical characteristics of ionospheric clutter can typically be approximated by a Gaussian distribution. The normal distribution matrix can effectively capture these characteristics of ionospheric clutter, thereby more accurately describing its covariance structure.

Table 1. Signal-to-Clutter ratio

DOA	MVDR	M-MVDR	N-M-MVDR	U-M-MVDR	I-M-MVDR	A-M-VDR
23.2297	24.4647	27.5088	27.7199	27.4809	27.4530	27.2864

4. Summary

The MVDR algorithm requires using the MLE method to estimate the covariance matrix of the ionospheric clutter. MLE requires all samples to satisfy the condition of being independently and identically distributed (i.i.d.). However, ionospheric clutter samples clearly do not meet this requirement. Therefore, this paper uses the Mahalanobis distance method to select samples that satisfy the i.i.d. condition. Since ionospheric clutter is non-stationary over time, it presents numerical stability issues. This paper introduces four different forms of loading factors. Simulation results show that the loading factor in the form of a normal distribution provides the best suppression of ionospheric clutter.

References

[1] RAVAN M, RIDDOLLS R J, and ADVE R S. Ionospheric and auroral clutter models for HF surface wave and over-the horizon radar systems[J]. Radio Science, 2016, 47(3): 1-12.

[2] Zhang X, Yang Q, Yao D, et al. Main-Lobe Cancellation of the Space Spread Clutter for Target Detection in HFSWR[J]. IEEE Journal of Selected Topics in Signal Processing, 2017, 9(8):1632-1638.

[3] Lei L, Xu R, Li G. Robust Adaptive Beamforming Based on eralized Sidelobe Cancellation[C]// International Conference on Radar. IEEE, 2006:1-4.

[4] Mao T, Xia W M, Cui-Ping Q U, et al. A Study on Characteristics and Applications of HFGround Wave OTH Radar[J]. Modern Radar, 2009, 31(3):7-10.

[5] Ma Kunlong. Short term distributed load forecasting method based on big data. Changsha: Hunan University, 2014.

[6] CHEN S Y, GILL E W, and HUANG W M. A high-frequency surface wave radar ionospheric clutter model for mixed-path propagation with the second-order sea scatter[J]. IEEE Transactions on Antennas and Propagation, 2016, 64(12): 5373-5381.

[7] PONSFORD A M and WANG J. A review of high frequency surface wave radar for detection and tracking of ships[J]. Turkish Journal of Electrical Engineering and Computer Sciences, 2010, 18(3): 409-428.

[8] JI Y G, ZHANG J, WANG Y M, et al. vessel target detection based on fusion range-Doppler image for dual-frequency hignfrequency[J]. IET Radar Sonar and Navigation, 2016, 10(2): 333-340.

[9] SALEH O, RAVAN M, RIDDOLLS R, et al. Fast fully adaptive processing: A multistage STAP approach[J]. IEEE Transactions on Aerospace and Electronic Systems, 2016, 52(5): 2168-2183.

[10] Su Y, Wei Y, Xu R, et al. Ionospheric clutter suppression using Wavelet Oblique Projecting Filter[C]// Radar Conference. IEEE, 2017:1552-1556.

[11] Jangal F, Saillant S, Helier M. Ionospheric clutter cancellation and wavelet analysis[C]// European Conference on Antennas and Propagation. IEEE, 2006:1-6.

[12] WU M, WEN B Y, and ZHOU H. Ionospheric clutter suppression in HF surface wave radar[J]. Journal of Electromagnetic Waves and Applications, 2012, 23(10): 5-1272.