Determination of Gauss-Krüger Projection Parameters with Minimizing Maximum Projection Deformation for Large-Scale Engineering

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Abstract. Currently, engineering construction is placing increasing demands on the accuracy of engineering measurement. Projection deformation has a significant effect on engineering measurement accuracy, representing a significant challenge in the field of high-precision measurement. In the case of Gauss-Krüger projection of arbitrary zone, setting the projection reference position in the center of the survey area will not necessarily guarantee the minimum possible projection deformation. This article presents a method for determining the projection reference position of Gauss-Krüger projection based on the minimum standard of the maximum projection deformation, along with the maximum applicable range of the projection parameter. Results demonstrate lower absolute values of length differences, smaller standard deviations of side length differences, and superior overall results. In comparison to existing methods, the proposed method has a broader scope of application.

Keywords: Gauss-Krüger projection of arbitrary zone; minimize maximum projection deformation; projection reference position; maximum projection range; large-scale precision measurement

1. Introduction

In light of the evolving global landscape and the robust growth of China's national economy, the scope of modern engineering construction has been gradually expanding, accompanied by the rising complexity of engineering structures. This situation has introduced new challenges in ensuring the quality of precision engineering measurement results and controlling projection deformation, which has long been a significant concern in the imputation of various types of precision engineering measurement data. The question of how to control projection deformation in such areas to improve the positioning accuracy and operational efficiency of engineering measurement represents a significant challenge for engineering applications [1].

A multitude of factors influence the accuracy of measurements and the expression of geographic information data [2]. Among these, the loss of accuracy due to length deformation caused by map projection has a significant impact on the precision engineering coordinate system. High accuracy in engineering construction can be achieved by adopting appropriate projection methods to ensure that the overall projection deformation meets the specified requirements and achieves higher accuracy.

The horizontal distance of the measurement edge within the measurement area is designated as d. In the projection of the measurement edge, a length deformation is generated, which is recorded as Δd. This approach consists of two main parts: First, it is projected to the reference ellipsoid, recorded as d_1, and this process generates a length deformation due to elevation difference, recorded as Δd_1. Subsequently, the measurement edge is projected from the reference ellipsoid to the Gaussian surface, recorded as d_2. This process generates a length deformation due to the Gauss-Krüger projection, which is denoted as Δd_2. From the aforementioned projection process, the edge length of the measurement edge obtained by projecting the measurement edge to the reference ellipsoid is $d_1 = d + \Delta d_1$, and the edge length of the ranging edge obtained by projecting from the reference ellipsoid to the Gaussian surface is $d_2 = d_1 + \Delta d_2$ [3].

$$\Delta d = \Delta d_1 + \Delta d_2 = \frac{d}{2r^2}(y_m^2 - 2rh_m) \quad (1)$$

where Δd represents the total deformation of the measurement edge in the process of projection onto the Gaussian surface; r refers to the mean radius of curvature at the center of the selected measurement edge, which in practice is often replaced by the mean radius of curvature of the Earth, generally assumed to be 6371 km; y_m represents the mean value of the transverse coordinates of the two endpoints of the selected measurement edge; and h_m is the mean elevation of the measurement edge projection datum plane with respect to the reference ellipsoidal surface.

In engineering surveys, projection parameters that are suitable for the characteristics of the project need to be selected for projects that have strict limitations on the maximum projected deformation value of the length. The traditional practice is usually to take the center of the survey area as the reference position, but this method often fails to ensure that the maximum projected deformation value meets the strict requirements of engineering construction. Therefore, studying a method of Gauss-Krüger projection of arbitrary zone based on the minimum criterion of maximum projected deformation is particularly important.

Through detailed analysis and calculation, this paper not only determines the reference position where the maximum projected deformation is minimized, but also defines the range of application of the parameters that satisfy the condition of the maximum permissible deformation limit of the projected length per kilometer. This research provides an accurate and reliable engineering measurement method that helps satisfy the implementation of engineering projects with extremely high measurement accuracy requirements.

Fig. 1 illustrates a diagram of the reference position of Gauss-Krüger projection of the arbitrary zone. In this diagram, Δl_E represents the distance from the reference position to the easternmost side of the survey area, while Δl_W denotes the distance from the reference position to the westernmost side of the survey area.

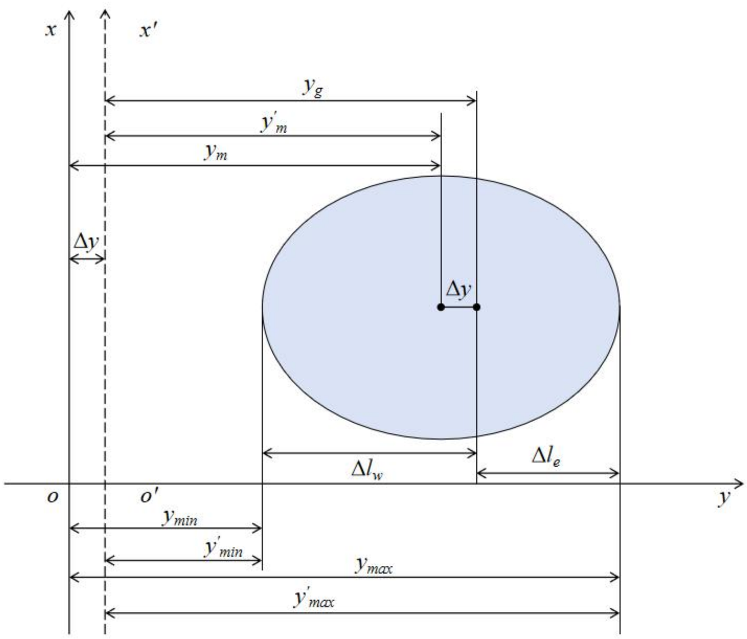

Fig. 1 Diagram of the position of the reference center of gravity for Gauss-Krüger projection of arbitrary zone

2. Determination of Gauss-Krüger Projection Parameters with Minimizing Maximum Projection Deformation

2.1 Determination of the distance from the central meridian to the projected reference position of the survey area

The deformation of the horizontal distance projected on the Gaussian surface Δd=0 can be guaranteed by calculating the distance between the central meridian and the projected reference position of the survey area $y_m^{'}$. This relationship can be demonstrated by the following equation:

$$d\frac{y_m^{'2}}{2r^2} = d\frac{h_m}{r} \tag{2}$$

The distance between the new central meridian and the projected reference position of the survey area $y_m^{'}$ can be calculated using the aforementioned formula (2). The following equation illustrates this calculation:

$$y_m^{'} = \sqrt{2rh_m} \tag{3}$$

The distance from the new central meridian to the projected reference position of the survey area $y_m^{'}$ can then be determined.

2.2 Determination of the reference position for the projection of the survey area

The determination of the distance between the central meridian and the projected reference position of the survey area $y_m^{'}$ in terms of minimizing the value of maximum projected deformation is a pivotal step in the proposed method. Once the distance has been determined, the next step is to accurately locate the survey area reference position y_g to ensure that the maximum projected deformation limits set out in the engineering specifications are met. However, the traditional approach of using the center of the survey area as the reference location is often inadequate for ensuring that the maximum projected deformation is minimized because the edge deformation after Gauss-Krüger projection shows a quadratic trend along the transverse axis, and simply locating the reference position in the center of the survey area does not effectively control the maximum value of this deformation. Consequently, a more refined method for determining the reference position of the survey area needs to be used to minimize the maximum projected deformation of the survey area. This method should be based on the criterion of minimizing the value of the maximum projected deformation of the survey area.

The maximum projected deformation $\max(\Delta S_i) = \max(\Delta D_e, \Delta D_w)$ is minimized, where ΔD_e and ΔD_w denote the maximum Gauss-Krüger projected deformation at the east and west ends of the survey area, respectively. For Gauss-Krüger projection of arbitrary zone with a minimal maximum projected deformation, the survey area reference position that satisfies $\Delta D_e = \Delta D_w$ needs to be determined based on the obtained $y_m^{'}$.

The maximum transverse coordinate value y_{max} and the minimum transverse coordinate value y_{min} before the meridian movement are known; therefore, the east–west span of the survey area can be calculated as $l = y_{max} - y_{min}$. Subsequently, the distance from the reference position to the westernmost side of the survey area Δl_w and the distance from the reference position to the easternmost side of the survey area Δl_e are determined in accordance with the distance between the central meridian and the projected reference position of the survey area $y_m^{'}$ and the east–west span of the survey area l. Setting the deformation of positions $y_{max}^{'}$ and $y_{min}^{'}$ equal, that is, $\Delta D_e = \Delta D_w$, the following equation can be obtained:

$$d_1\frac{y_m^{'2}}{2r^2} - d_1\frac{y_{min}^{'2}}{2r^2} = d_1\frac{y_{max}^{'2}}{2r^2} - d_1\frac{y_m^{'2}}{2r^2} \tag{4}$$

where $y_{max}^{'}$ and $y_{min}^{'}$ represent the maximum and minimum transverse coordinate values of the survey area, respectively, after the central meridian has been moved.

Setting the distance from the original survey area projection reference position $y_m^{'}$ to the easternmost side of the survey area as Δl_e, the maximum value of the survey area transverse coordinate after moving the meridian can be calculated by $y'_{max} = y'_m + \Delta l_e$. Similarly, the minimum value of the survey area transverse coordinate after moving the meridian can be calculated by $y'_{min} = y'_m + \Delta l_e - l$. Substituting these values into the above equation yields the following form:

$$2\Delta l_e^2 + 2\left(2y'_m - l\right)\Delta l_e - \left(2y'_m - l\right)l = 0 \tag{5}$$

Given that Δl_e is required to be positive, it can be solved as shown below

$$\Delta l_e = \frac{\sqrt{\left(2y'_m - l\right)^2 + 2\left(2y'_m - l\right)l} - \left(2y'_m - l\right)}{2} \tag{6}$$

From this, the maximum and minimum values of the transverse coordinates of the survey area in the new projection zone can be determined.

Then, we can ascertain the value of the change in the horizontal coordinates of the old and new coordinates, which can be expressed as $\Delta y = y'_{max} - y_{max}$. Consequently, we can determine the new projected reference position y_g, as shown below

$$y_g = y'_m + \Delta y \tag{7}$$

2.3 Determination of the elevation of the compensation projection plane

The offset value of the Gauss-Krüger projected deformation of the measurement edge is calculated below. For Δd =0 at the point y_g, the following equation is obtained [4]:

$$h'_m = \frac{y_g^2}{2r} \tag{8}$$

where h'_m represents the Gauss-Krüger deformation compensation value. The formula for the elevation of the projection datum plane with compensation effect can be derived as follows:

$$h = h_m - h'_m \tag{9}$$

where h represents the elevation of the projection datum plane with compensation effect.

2.4 Gauss-Krüger projection transformation

The geodetic longitude of the new central meridian M can be calculated by the inverse solution of the geodetic problem [5].

Setting the longitude of the central meridian to M and applying reprojection to transform the coordinates of the survey area under the plane coordinate system corresponding to the new central meridian can realize Gauss-Krüger projection of arbitrary zone based on the minimum criterion of maximum projected deformation [6].

The projection parameters calculated by the proposed method can theoretically ensure that the projection deformation of the easternmost and westernmost sides of the survey area is minimized, which can easily meet the needs of projects with high requirements for the maximum projection deformation.

2.5 Maximum applicable range of selected projection parameters

Once the projection parameters have been selected, the maximum range of application of the east and west edges of the survey area needs to be evaluated further. Doing so ensures that the limit of projected deformation is met and that the projected deformation in the survey area is controlled within the permissible limit.

With lim_e denoting the maximum range of application allowed east of the projected reference position, lim_w indicating the maximum range of application allowed west, and Δlim_d representing the maximum allowable deformation per kilometer, the formulas for calculating lim_e and lim_w can be obtained as follows:

$$lim_e = \sqrt{2r^2 \frac{\Delta lim_d}{d_1} + y'^2_m} - y'_m \tag{10}$$

$$lim_w = y'_m - \sqrt{y'^2_m - 2r^2 \frac{\Delta lim_d}{d_1}} \tag{11}$$

So far, the maximum range of applicability of the parameters of the proposed method has been determined.

The range of compensation projection should be maintained within $[y_w, y_e]$, which is the specified range of $\left[y_m^{'} - lim_w, y_m^{'} + lim_e\right]$, to ensure that the maximum projected deformation complies with the limitations specified in the code or project.

3. Example Analysis

The deformation after compensation projection with the parameters determined by $y_m^{'} = 116\ km$ is listed in Fig. 2.

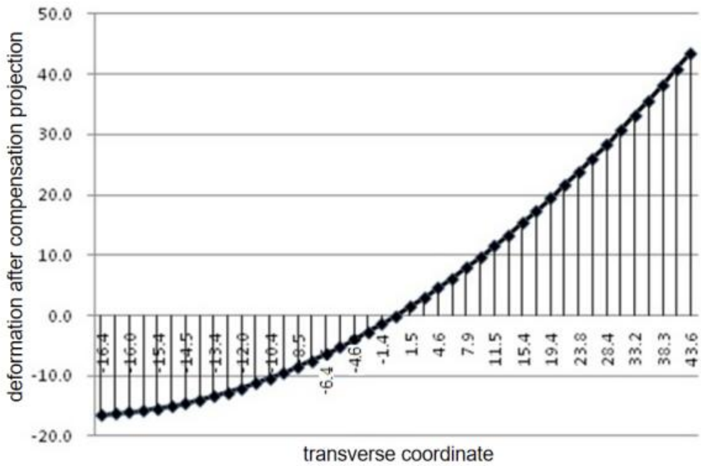

Fig. 2 Diagram of deformation after compensation projection at $y_m^{'} = 116\ km$

As illustrated in Fig. 2, the projection deformation is precisely zero at the selected compensation projection position of $116\ \mathrm{km}$ from the central meridian. This finding is of paramount importance because it indicates that at that projection location, the projection does not exhibit length deformation. On the basis of the maximum allowable deformation per kilometer specified in the code (e.g., $\Delta lim_d = 2.5\ cm$ in the Standard for Engineering Surveys), the result is shown below

$$lim_e = \sqrt{2r^2\frac{\Delta lim_d}{d_1} + y_m^{'2}} - y_m^{'} = 8441\ m$$

$$lim_w = y_m^{'} - \sqrt{y_m^{'2} - 2r^2\frac{\Delta lim_d}{d_1}} = 910\ m$$

$$y_w = y_m^{'} - lim_w = 106895\ m$$

$$y_e = y_m^{'} + lim_e = 124441\ m$$

The preceding results indicate that the maximum allowable span for the project to satisfy the projected deformation limit condition is $lim_e + lim_w = 17.55\ km$.

The proposed Gauss-Krüger projection of arbitrary zone with minimizing maximum projection deformation demonstrates notable advantages in regulating the maximum projected length deformation of the Gauss-Krüger projection of arbitrary zone because of its inherent property of minimizing the maximum projected deformation.

In comparison to the conventional GNSS observation, the precision ranging technique exhibits greater accuracy in edge length measurement. The results of experimental calculations of specific engineering projects are compared and analyzed with the results of precision ranging to verify the accuracy of the proposed method. This comparison allows for the evaluation of the difference in accuracy between the traditional method and the proposed method. The specific comparison results between the traditional method and the proposed method are presented in Table 1, and the statistical charts of the difference in edge length between the traditional method and the proposed method are shown in Figs. 3 and 4. These charts provide a detailed analysis of the performance and accuracy differences between the two methods in practical applications, thus serving as an important reference basis for engineering practice.

Edge number	Precision ranging/m	Traditional method/m	Difference with precision ranging/mm	Proposed method/m	Difference with precision ranging/mm
1	804.855	804.840	−15.0	804.830	−24.8
2	683.072	683.081	9.0	683.073	1.1
3	860.255	860.279	24.0	860.268	12.8
4	524.753	524.759	5.8	524.755	2.2
5	818.368	818.379	10.7	818.371	3.1
6	797.997	798.024	26.8	798.012	15.3
7	1075.334	1075.349	15.2	1075.337	3.3
8	618.516	618.536	19.9	618.530	13.6

Table 1. Comparison of edge lengths of the proposed method relative to traditional methods

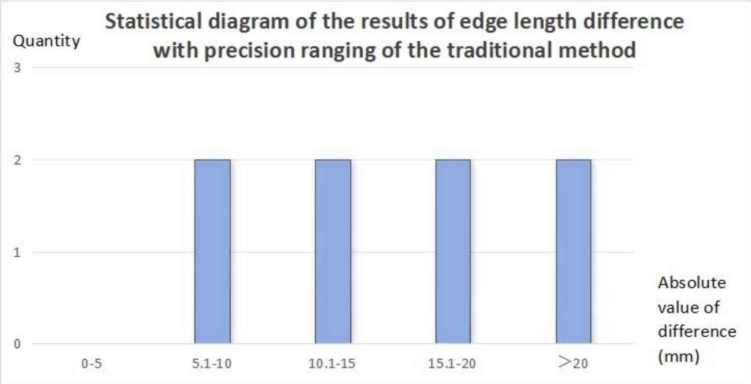

Fig. 3 Statistical diagram of the results of edge length difference with precision ranging of the traditional method

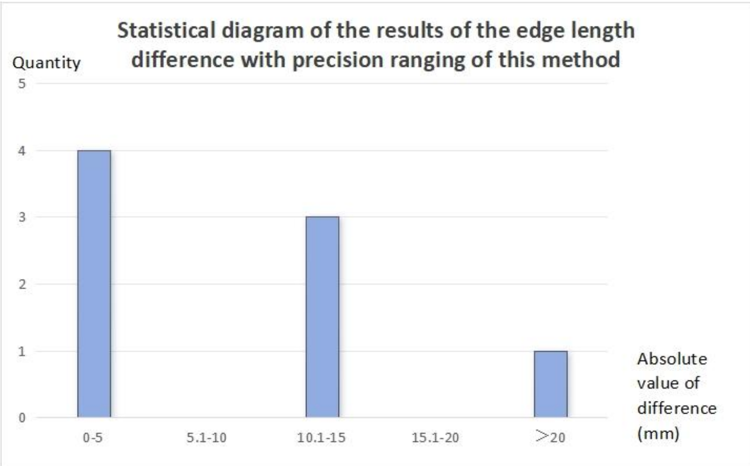

Fig. 4 Statistical diagram of the results of the edge length difference with precision ranging of this method

The results in Table 1 show that when calculated by the traditional method, the mean value of the difference in edge length is 12.0 mm, the mean value of the absolute value of the difference in edge length is 15.8 mm, and the standard deviation of the difference in edge length is $\pm 11.6\ mm$. When calculated by the proposed method, the mean value of the difference in edge length is 3.3 mm, the mean value of the absolute value of the difference in edge length is 9.5 mm, and the standard deviation of the difference in edge length is $\pm 11.2\ mm$.

In conclusion, the absolute value of edge length difference, average value of edge length difference, average value of absolute value of edge length difference, and standard deviation of edge length difference obtained by the proposed method are lower, and the comprehensive effect is better.

4. Summary

The reasonable selection of parameters of Gauss-Krüger projection of arbitrary zone is crucial in the process of exploring methods for reducing the projection deformation of the survey area through the use of Gauss-Krüger projection of arbitrary zone. This paper presents a parameter selection method with the objective of minimizing the maximum projection deformation of Gauss-Krüger projection of arbitrary zone.

In this paper, the distance between the new central meridian and the projected reference position of the survey area $y_m^{'}$ is first determined under the premise of minimizing the maximum projection deformation. Second, with the use of the known distance from the central meridian to the projected reference position of the survey area and the east–west span of the survey area, the distance from the reference position to the westernmost side of the survey area and its distance to the easternmost side of the survey area are calculated, and then the new projected reference position y_g is accurately located. Then, on the basis of the new projection reference position y_g, the distance between the old and new central meridian is calculated, and the longitude of the new central meridian M is determined by the inverse solution of the geodetic problem. Finally, with the use of the Gauss-Krüger projection transformation method, the coordinates of the survey area are transformed to the new central meridian under Gauss-Krüger projection of arbitrary zone, and reprojection calculation is performed to obtain the coordinates under the new central meridian of the survey area and establish the corresponding coordinate conversion relation equation. In addition, this paper defines the maximum applicable range of the selected projection parameters to ensure the applicability of the method.

Compared with the methods used in current engineering practice, the proposed method not only minimizes the maximum projection deformation under the same conditions, but also has a wider application scope in the same projection zone. The proposed method provides an efficient and accurate solution for engineering measurement, which helps improve the accuracy and quality of engineering construction.

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